2020

B.Sc-part-II (Sub) Diff. Calculus Date. 03-07-2020

Problems based on Derivatives of Higher order.

Q.1. If $y = \log(\sin x)$, show that $y_3 = \frac{2\cos x}{\sin^3 x}$

Soln: $\because y = \log(\sin x)$

Differentiating w.r. to x, we get

$y_1 = \frac{1}{\sin x}\cos x = \cot x$

Again, diff. w.r. to x, we get

$y_2 = -\operatorname{cosec}^2 x$

Again, diff. w.r.t. x, we get

$y_3 = -2\operatorname{cosec} x\,(-\operatorname{cosec} x \cdot \cot x)$

$= 2\operatorname{cosec}^2 x \cdot \cot x$

$\Rightarrow y_3 = \frac{2}{\sin^2 x} \cdot \frac{\cos x}{\sin x} = \frac{2\cos x}{\sin^3 x}$ Proved.

Q. 2. If $y = ax^{n+1} + bx^{-n}$, prove that

$x^2 y_2 = n(n+1)$.

Soln: We have, $y = ax^{n+1} + bx^{-n}$

Diff. with respect to x, we get

$y_1 = a(n+1)x^{n} - bn\,x^{-(n+1)}$

Again, diff. w.r.t. x, we get

$y_2 = an(n+1)\,x^{n-1} + bn(n+1)\,x^{-(n+2)}$

$\Rightarrow x^2 y_2 = an(n+1)\,x^{n+1} + bn(n+1)\,x^{-n}$

$= n(n+1)\,(ax^{n+1} + bx^{-n}) = n(n+1)y$

Hence, $x^2 y_2 = n(n+1)y$

Proved.

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Q.3 If $y = Ae^{mx} + Be^{-mx}$, prove that $y_2 = m^2y$.

Soln.: $\because y = Ae^{mx} + Be^{-mx}$

Diff. w.r. to x, we get

$y_1 = Ame^{mx} - Bme^{-mx}$

Again, diff. w.r. to x, we get

$y_2 = Am^2e^{mx} + Bm^2e^{-mx}$

$= m^2(Ae^{mx} + Be^{-mx})$

Hence, $y_2 = m^2y$ Proved.

Q.4. If $y = A\sin mx + B\cos mx$, prove that $y_2 = -m^2y$

Soln.: $\because y = A\sin mx + B\cos mx$.

Diff. w.r. to x, we get

$y_1 = Am\cos mx - Bm\sin mx$

Again, diff. w.r. to x, we get

$y_2 = -Am^2\sin mx - Bm^2\cos mx$

$= -m^2(A\sin mx + B\cos mx)$

$\therefore y_2 = -m^2y$ Proved.